NS – 607

I Semester B.C.A. Degree Examination, November/December 2016
(CBCS) (F+R)
(2014-15 & Onwards)
BCA – 105 : DISCRETE MATHEMATICS

Time : 3 Hours Max. Marks : 100

***Instruction** : Answer **all** Sections.*

SECTION – A

I. Answer **any ten** : **(10×2=20)**

1) If $A = \{x | x \in N \text{ and } x<3\}$ and $B = \{0, 1, 3\}$. Find $A - B$.

2) If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ and $C = \{0, 2, 3\}$, find $(A \cap B) \times C$.

3) Construct truth table for the proposition $p \vee \sim q$.

4) Find x, y, z if $\begin{bmatrix} 4-y & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} -1 & z+1 \\ 1 & 5 \end{bmatrix}$.

5) If $A = \begin{bmatrix} 1 & -2 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 3 \\ 3 & 1 & 4 \end{bmatrix}$, find AB.

6) Find the characteristic equation of the matrix $\begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$.

7) Prove that $\log_b a . \log_c b . \log_a c = 1$.

8) Find n if $2\,({}^nP_3) = {}^nP_5$.

9) On the set of integers Z, the binary operation $*$ is defined by

$a*b = \frac{ab}{3}, \forall a, b \in Z$. Find identity element.

10) If $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ find unit vector along $\vec{a} - \vec{b}$.

P.T.O.

11) Find the midpoint of line joining (–2, 8) and (1, – 2).

12) Find the equation of the line passing through (–1, 2) and having slope 3.

SECTION – B

II. Answer **any six** of the following : **(6×5 =30)**

13) If A = {1, 4}, B = {2, 3, 6}, C = {2, 3, 7} then verify that $A\times(B-C) = (A\times B) - (A\times C)$.

14) Show that the function $f : R \to R$ defined by $f(x) = 4x + 3$ is invertible. Find the inverse of f.

15) Show that $p \vee (q \wedge r) \leftrightarrow [(p\vee q)\wedge(p \vee r)]$ is a tautology.

16) If $(p \to q) \wedge (p\wedge r)$ is given to be false, find the truth values of p, q, r.

17) Write the truth table of $(p\vee q)\vee \sim p$. Show that the compound propositions $p\wedge q$ and $\sim(p\to \sim q)$ are logically equivalent.

18) Find the inverse of the matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$.

19) Using Cramer's rule solve $3x - y + 2z = 13$; $2x + y - z = 3$; $x + 3y - 5z = -8$.

20) Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$.

SECTION – C

III. Answer **any six** of the following. **(6×5 =30)**

21) If $\log\left(\frac{a-b}{5}\right) = \frac{1}{2}(\log a + \log b)$, show that $a^2 + b^2 = 27$ ab.

22) Find the number of three digit even numbers that can be formed using 2, 3, 4, 5, 6 repetitions being not allowed.

23) If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$ find n.

24) Prove that the set $G = \{ 3n \mid n \in Z\}$ is an abelian group w.r.t. addition.

25) Prove that the set $G = \{2, 4, 6, 8\}$ is an abelian group w.r.t. multiplication modulo 10.

26) If $\bar{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ find $(\bar{a} + 2\bar{b}).(2\bar{a} - \bar{b})$.

27) Show that the points A(1,2,3), B (2, 3, 1) and C (3,1,2) are vertices of an equilateral triangle.

28) If the vectors $4\hat{i} + 11\hat{j} + m\hat{k}$, $7\hat{i} + 2\hat{j} + 6\hat{k}$ and $\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar, then find 'm'.

SECTION – D

IV. Answer **any four** of the following. **(4×5 =20)**

29) Prove that the points (6, 4), (7, – 2), (5, 1), (4, 7) form vertices of a parallelogram.

30) The three vertices of a parallelogram taken in order are (8,5), (–7, –5) and (– 5, 5). Find the co-ordinate of the fourth vertex.

31) Find the equation of the locus of a point which moves such that its distance from X-axis is twice its distance from Y-axis.

32) Derive the equation of the straight line whose x -intercept is 'a' and y-intercept is 'b'.

33) Find 'K' for which the lines $2x - ky + 1 = 0$ and $x + (k+1) y - 1 = 0$ are perpendicular.

34) Find the equation of straight line which is passing through intersection of the lines $2x - 3y - 4 = 0$ and $2x + 2y - 1 = 0$ and perpendicular to the line $x + 4y - 8 = 0$.
